Steady State Pathology of Simple Regulating Systems

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ABSTRACT

Long-duration deviations from the original steady state are found in many diseases. An analysis of the steady state behaviour of simple regulating systems is, hence, of value to understand the underlying mechanisms.

For a simple model at least four classes of diseases can be distinguished: 'non-regulated state' diseases, 'reference' diseases, 'feedback gain' diseases and 'forward gain' diseases. For each class of diseases its effects can be deduced from the regulation characteristic, because of the differences in behaviour between the two pivotal points in the graph with regard to each class of diseases, and - into detail - from the block diagram models of the regulating systems. One also has to keep in mind the differences between systems regulating against excess (right-regulating systems) and against loss (left-regulating systems).

1. DISEASES OF THE STEADY STATE

Many diseases are characterised by a long duration change of some variable(s), out of the normal range of values. Hypertension, hyperthyroidy, hypothyroidy, diabetes mellitus (with too low levels of insulin and too high blood-glucose levels) are familiar examples. The average levels are either too high (a hyper-state) or too low (a hypo-state of the variable under consideration) and they remain elevated or lowered. The mentioned examples concern regulating systems (blood pressure regulation, the system regulating the bloodplasm level of thyroid hormones, and the insulin-loop in blood glucose regulation, respectively). For a given loop a steady state change of the level of the regulated variable may be caused by many different diseases of the system components. It is, hence, of interest to analyse a simple regulating system for long-term changes of the regulated variable in relation to diseases of the different parts of such a loop.

2. LEFT- AND RIGHT-REGULATING SYSTEMS

For simple biological feedback systems we have to distinguish two cases (Fig. 1): systems regulating against losses (left-regulating systems: LRS) and those working against excess levels of the regulated variable y_r (right-regulating systems: RRS) /2/.

From the block diagrams of these systems (Fig. 2), the equation for the steady state value of the regulated variable is for both systems given by:

$$y_r = \frac{K}{1 + K} y_i + \frac{1}{1 + K} y_n \qquad (1)$$

with the (open loop) gain K:

$$K = K_f K_c, \qquad (2)$$

and the critical state y_i:

$$y_i = \frac{x_i}{K_f} \qquad (3)$$

Here K_f is the feedback gain factor (of the sensor), K_c is the forward gain (of the corrector), x_i is the reference and y_n is the non-regulated state (the set of values for y_r in the absence of correction y_c: the line $y_r = y_c$ in Fig. 1).

The effect of the sensor gain (3) is of much importance and may, therefore, not be neglected. Note that the term 'set-point' is not used in this paper, since its meaning is often not quite clear, which may lead to confusion between the reference x_i and the critical state y_i (cf./1/ for a discussion of problems related to the direct application of classical regulation theory to biological feedback systems).

3. DIFFERENCES BETWEEN LRS AND RRS

The first set of steady diseases -non-regulated state diseases - is given by saturation under extreme loads. The regulating system is then either overloaded, or underloaded (Fig. 1). The ranges of under- and overload are reversed for a RRS

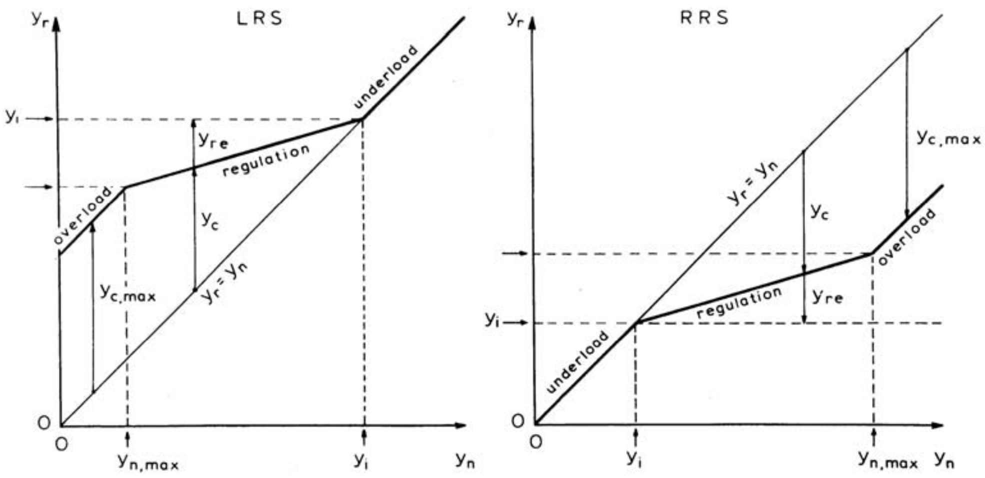

Figure 1. Linearized steady state characteristics for LRS (left) and RRS (right). Regulated state y_r, non-regulated state y_n, ideal state y, correction yc, maximal correction $y_{c,max}$, remaining error y_{re} . Arrows y_i: central critical point; unmarked arrows and arrows $y_{n,max}$: peripheral critical point. From /2/.

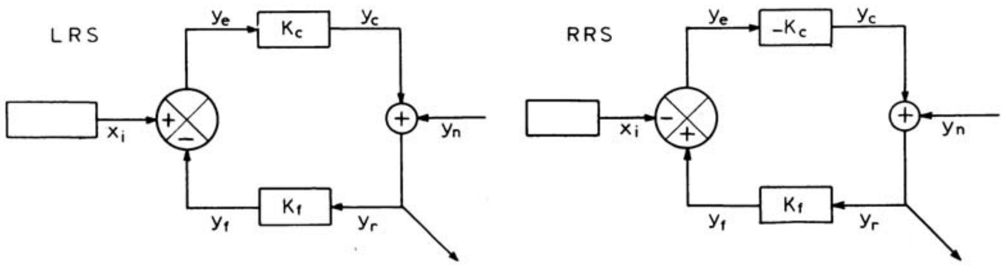

Figure 2. Block-diagrams for LRS (left) and RRS (right). Feedback signal y_f, feedback gain K_f, reference x_i, error signal y_e, Correction y_c, forward gain K_c. From /2/.

with respect to a LRS (Fig. 1).

The difference between RRS and LRS also becomes visible upon a decrease of the feedback gain K or, in the extreme situation, a disruption of the feedback loop (the feedback signal y_f becomes zero). Then a LRS becomes maximally active (leading to exhaustion), while a RRS ceases all activity (is switched off). In the case of debilitating oscillating diseases of regulating systems, disruption of the feedback loop may be seriously considered when the system is a RRS. This applies, for instance, to grave muscular tremors in neurological diseases.

4. DISEASES

To understand the steady state pathology (hyper- or hypofunction), resulting from malfunction of system components, equations 2 and 3 are of importance, together with the equation for the crossing between the (extrapolated) line for regulation and the abscissa:

$$y_n = -K_c x_i \text{ for } y_r = 0 \ . \tag{4}$$

The points (y_n,y_r) given by (3) and by (4) are, respectively, $(-K_c x_i, 0)$ on the abscissa and (y_i,y_i) on he line $y_r = y_n$ (Fig. 3). The effect of changes in the sensor gain K_f, or in the forward gain K_c or in the reference x_i lead to different shifts of these two points and, hence, of the line for regulation. The resulting effects upon the steady state size of signals and states can be read from the

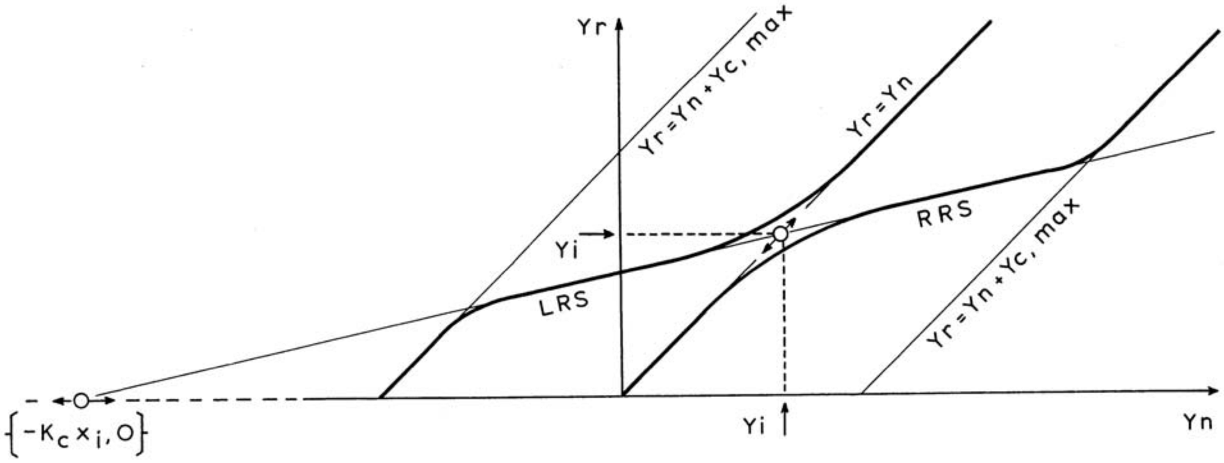

Figure 3. The two points (circles) which determine the position of the regulation line. Possible shifts are indicated by heavy arrows.

graphs (Fig. 1,3) together with a simple calculation from the block diagrams (Fig. 2), starting with the diseased element. A tabulation of the shifts for both LRS and RRS will be quite helpful in the differential diagnosis for steady state diseases of simple feedback systems.

For a reference disease (x_i) the line shifts up or down, in parallel to the original one, for an increase or a decrease of x_i respectively.

The point (y_i,y_i) does not change in position upon a change of the forward gain K_c, while the point on the abscissa shifts to the left upon an increase and to the right upon a decrease of K_c. The effect is simply a change of the (open loop) gain K (2), visible by a change of the slope of the line through (y_i,y_i).

Changes of the feedback gain K_f or feedback signal y_f profoundly affect the regulation. The abscissal point does not change in position, while the (y_i,y_i) point shifts upwards along the line y_r = y_n for a decrease of K_f, and downwards for an increase. An increase of K_f decreases the ideal state y. (3) and increases the open loop gain (2). The system is, hence, clamped onto a lower state (a hypo-state). A decrease of K_f increases the ideal state y_f (3) and weakens the system (2). For a LRS the correction y_c increases and may lead to overload and exhaustion, while for a RRS the correction decreases, leading to underload of the system which 'switches off'.

Secondary effects of steady state diseases may generate hypertrophy (increase of the gain of elements) or atrophy (gain decrease), due, respectively, to an increase or decrease of the average input of the elements under consideration. These effects may counteract the effects of the original disease, but the compensation (when present) is not complete. A detailed discussion of the properties of diseased regulatory systems in the steady state is in preparation.

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